

Interplay between superconductivity and pseudogap state in bilayer cuprate superconductors

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The interplay between the superconducting gap and normal-state pseudogap in the bilayer cuprate superconductors is studied based on the kinetic energy driven superconducting mechanism. It is shown that the charge carrier interaction directly from the *interlayer* coherent hopping in the kinetic energy by exchanging spin excitations does not provide the contribution to the normal-state pseudogap in the particle-hole channel and superconducting gap in the particle-particle channel, while only the charge carrier interaction directly from the *intralayer* hopping in the kinetic energy by exchanging spin excitations induces the normal-state pseudogap in the particle-hole channel and superconducting gap in the particle-particle channel, and then the two-gap behavior is a universal feature for the single layer and bilayer cuprate superconductors.

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The conventional superconductors are characterized by the energy gap, which exists in the excitation spectrum below the superconducting (SC) transition temperature T_c , and therefore is corresponding to the energy for breaking a Cooper pair of the charge carriers and creating two excited states¹. However, in the cuprate superconductors, an energy gap called the normal-state pseudogap exists^{2,3} above T_c but below the pseudogap crossover temperature T^* , which is associated with some anomalous properties. Although the charge carrier pair gap in the cuprate superconductors has a dome-like shape of the doping dependence^{4,5}, the magnitude of the normal-state pseudogap is much larger than that of the charge carrier pair gap in the underdoped regime^{2,3}, then it smoothly decreases upon increasing doping, and seems to merge with the charge carrier pair gap in the overdoped regime, eventually disappearing together with superconductivity at the end of the SC dome². In this case, the charge carrier pair gap and normal-state pseudogap are thus two fundamental parameters of the cuprate superconductors whose variation as a function of doping and temperature provides important information crucial to understanding the details of superconductivity^{2,3}.

Experimentally, a large body of experimental data obtained by using different measurement techniques have provided rather detailed information on the low-energy excitations of the single layer and bilayer cuprate superconductors²⁻⁵, where the Bogoliubov-quasiparticle nature of the low-energy excitations is unambiguously established⁶. However, there are numerous anomalies for the bilayer cuprate superconductors^{4,5}, which complicate the physical properties of the low-energy excitations in the bilayer cuprate superconductors. This follows a fact that the bilayer splitting (BS) has been observed in the bilayer cuprate superconductors in a wide doping range⁷, which derives the low-energy excitation spectrum into the bonding and antibonding components due to the presence of the bilayer blocks in the unit cell. In particular, the well pronounced peak-dip-hump structure in the low-energy excitation spectrum of the bilayer cuprate superconductors has been attributed to

BS⁸⁻¹⁰. In this case, an important issue is whether the behavior of the normal-state pseudogap observed in the low-energy excitation spectrum as a suppression of the spectral weight is universal or not. Within the framework of the kinetic energy driven SC mechanism¹¹, the interplay between the SC gap and normal-state pseudogap in the single layer cuprate superconductors has been studied recently¹², where the interaction between charge carriers and spins directly from the kinetic energy by exchanging spin excitations induces the normal-state pseudogap state in the particle-hole channel and SC-state in the particle-particle channel, then there is a coexistence of the SC gap and normal-state pseudogap in the whole SC dome. In particular, this normal-state pseudogap is closely related to the quasiparticle coherent weight, and both the normal-state pseudogap and SC gap are dominated by one energy scale. In this paper, we study the interplay between the SC gap and normal-state pseudogap in the bilayer cuprate superconductors along with this line. We show explicitly that the weak charge carrier interaction directly from the *interlayer* coherent hopping in the kinetic energy by exchanging spin excitations does not provide the contribution to the normal-state pseudogap in the particle-hole channel and SC gap in the particle-particle channel, while only the strong charge carrier interaction directly from the *intralayer* hopping in the kinetic energy by exchanging spin excitations induces the normal-state pseudogap in the particle-hole channel and SC gap in the particle-particle channel, and then the two-gap behavior is a universal feature for the single layer and bilayer cuprate superconductors.

The single common feature in the layered crystal structure of the cuprate superconductors is the presence of the two-dimensional CuO_2 plane⁴, and then it is believed that the unconventional physics properties of the cuprate superconductors is closely related to the doped CuO_2 planes¹³. In this case, it is commonly accepted that the essential physics of the doped CuO_2 plane¹³ is captured by the t - J model on a square lattice. However, for discussions of the interplay between the SC gap and normal-state pseudogap in the bilayer cuprate supercon-

ductors, the t - J model can be extended by including the bilayer interaction as^{8,16},

$$H = -t \sum_{i\hat{\eta}a\sigma} C_{ia\sigma}^\dagger C_{i+\hat{\eta}a\sigma} + t' \sum_{i\hat{\tau}a\sigma} C_{ia\sigma}^\dagger C_{i+\hat{\tau}a\sigma} - \sum_{i\sigma} t_\perp(i) (C_{i1\sigma}^\dagger C_{i2\sigma} + H.c.) + \mu \sum_{ia\sigma} C_{ia\sigma}^\dagger C_{ia\sigma} + J \sum_{i\hat{\eta}a} \mathbf{S}_{ia} \cdot \mathbf{S}_{i+\hat{\eta}a} + J_\perp \sum_i \mathbf{S}_{i1} \cdot \mathbf{S}_{i2}, \quad (1)$$

supplemented by the local constraint $\sum_\sigma C_{ia\sigma}^\dagger C_{ia\sigma} \leq 1$ to remove double occupancy, where $a = 1, 2$ is plane index, the summation within the plane is over all sites i , and for each i , over its nearest-neighbors $\hat{\eta}$ or the next nearest-neighbors $\hat{\tau}$, $C_{ia\sigma}^\dagger$ and $C_{ia\sigma}$ are electron operators that respectively create and annihilate electrons with spin σ , $\mathbf{S}_i = (S_i^x, S_i^y, S_i^z)$ are spin operators, μ is the chemical potential, while the interlayer hopping has the form in the momentum space,

$$t_\perp(\mathbf{k}) = \frac{t_\perp}{4} (\cos k_x - \cos k_y)^2, \quad (2)$$

which describes coherent hopping between the CuO_2 planes. This functional form of the interlayer hopping (2) is predicted on the basis of the local density approximation calculations¹⁴, and later the experimental observed BS agrees well with it⁷. In this bilayer t - J model (1), the crucial requirement is to impose the electron single occupancy local constraint, which can be treated properly in analytical calculations within the charge-spin separation (CSS) fermion-spin theory^{15,16}, where the constrained electron operators are decoupled as $C_{ia\uparrow} = h_{ia\uparrow}^\dagger S_{ia}^-$ and $C_{ia\downarrow} = h_{ia\downarrow}^\dagger S_{ia}^+$, with the spinful fermion operator $h_{ia\sigma} = e^{-i\Phi_{ia\sigma}} h_{ia}$ that represents the charge degree of freedom together with some effects of the spin configuration rearrangements due to the presence of the doped hole itself (charge carrier), while the spin operator S_{ia} describes the spin degree of freedom, then the electron single occupancy local constraint is satisfied in analytical calculations. In this CSS fermion-spin representation, the bilayer t - J model (1) can be expressed as,

$$H = t \sum_{i\hat{\eta}a} (h_{i+\hat{\eta}a\uparrow}^\dagger h_{ia\uparrow} S_{ia}^+ S_{i+\hat{\eta}a}^- + h_{i+\hat{\eta}a\downarrow}^\dagger h_{ia\downarrow} S_{ia}^- S_{i+\hat{\eta}a}^+) - t' \sum_{i\hat{\tau}a} (h_{i+\hat{\tau}a\uparrow}^\dagger h_{ia\uparrow} S_{ia}^+ S_{i+\hat{\tau}a}^- + h_{i+\hat{\tau}a\downarrow}^\dagger h_{ia\downarrow} S_{ia}^- S_{i+\hat{\tau}a}^+) + \sum_i t_\perp(i) (h_{i2\uparrow}^\dagger h_{i1\uparrow} S_{i1}^+ S_{i2}^- + h_{i1\uparrow}^\dagger h_{i2\uparrow} S_{i2}^+ S_{i1}^- + h_{i2\downarrow}^\dagger h_{i1\downarrow} S_{i1}^- S_{i2}^+ + h_{i1\downarrow}^\dagger h_{i2\downarrow} S_{i2}^- S_{i1}^+) - \mu \sum_{ia\sigma} h_{ia\sigma}^\dagger h_{ia\sigma} + J_{\text{eff}} \sum_{i\hat{\eta}a} \mathbf{S}_{ia} \cdot \mathbf{S}_{i+\hat{\eta}a} + J_{\text{eff}\perp} \sum_i \mathbf{S}_{i1} \cdot \mathbf{S}_{i2}, \quad (3)$$

where $J_{\text{eff}} = J(1 - \delta)^2$, $J_{\text{eff}\perp} = J_\perp(1 - \delta)^2$, and $\delta = \langle h_{ia\sigma}^\dagger h_{ia\sigma} \rangle = \langle h_{ia}^\dagger h_{ia} \rangle$ is the doping concentration.

For the bilayer cuprate superconductors, there are two coupled CuO_2 planes in one unit cell. In this case, the SC order parameter for the electron Cooper pair is a

matrix⁸ $\Delta = \Delta_L + \sigma_x \Delta_T$, with Δ_L and Δ_T are the corresponding longitudinal and transverse parts, respectively. In the doped regime without an antiferromagnetic long-range order (AFLRO), the charge carriers move in the background of the disordered spin liquid state, and then the longitudinal and transverse SC order parameters can be expressed in the CSS fermion-spin representation as, $\Delta_L = -\chi_1 \Delta_{\text{hL}}$ and $\Delta_T = -\chi_\perp \Delta_{\text{hT}}$, with

$$\Delta_{\text{hL}} = \langle h_{i+\hat{\eta}a\downarrow} h_{ia\uparrow} - h_{i+\hat{\eta}a\uparrow} h_{ia\downarrow} \rangle, \quad (4a)$$

$$\Delta_{\text{hT}} = \langle h_{i2\downarrow} h_{i1\uparrow} - h_{i2\uparrow} h_{i1\downarrow} \rangle, \quad (4b)$$

are the corresponding longitudinal and transverse parts of the charge carrier pair gap parameter, respectively, and the spin correlation functions $\langle S_{ia}^+ S_{i+\hat{\eta}a}^- \rangle = \langle S_{ia}^- S_{i+\hat{\eta}a}^+ \rangle = \chi_1$ and $\langle S_{i1}^+ S_{i2}^- \rangle = \langle S_{i1}^- S_{i2}^+ \rangle = \chi_\perp$. The result in Eq. (4) shows that as in the single layer case¹¹, the SC gap parameter in the bilayer cuprate superconductors is also closely related to the corresponding charge carrier pair gap parameter, and therefore the essential physics in the SC-state is dominated by the corresponding one in the charge carrier pairing state.

Within the framework of the kinetic energy driven SC mechanism¹¹, the electronic structure of the bilayer cuprate superconductors has been discussed^{8,16}, and the result shows that the low-energy excitation spectrum is split into the bonding and antibonding components due to the presence of BS, then the observed peak-dip-hump structure is mainly caused by the bilayer splitting, with the peak being related to the antibonding component, and the hump being formed by the bonding component. Following our previous discussions^{8,16}, the self-consistent equations that satisfied by the full charge carrier normal and anomalous Green's functions are obtained as,

$$g(\mathbf{k}, \omega) = g^{(0)}(\mathbf{k}, \omega) + g^{(0)}(\mathbf{k}, \omega) [\Sigma_1^{(\text{h})}(\mathbf{k}, \omega) g(\mathbf{k}, \omega) - \Sigma_2^{(\text{h})}(-\mathbf{k}, -\omega) \Im^\dagger(\mathbf{k}, \omega)], \quad (5a)$$

$$\Im^\dagger(\mathbf{k}, \omega) = g^{(0)}(-\mathbf{k}, -\omega) [\Sigma_1^{(\text{h})}(-\mathbf{k}, -\omega) \Im^\dagger(-\mathbf{k}, -\omega) + \Sigma_2^{(\text{h})}(-\mathbf{k}, -\omega) g(\mathbf{k}, \omega)], \quad (5b)$$

respectively, where the full charge carrier normal Green's function $g(\mathbf{k}, \omega) = g_L(\mathbf{k}, \omega) + \sigma_x g_T(\mathbf{k}, \omega)$, the full charge carrier anomalous Green's function $\Im^\dagger(\mathbf{k}, \omega) = \Im_L^\dagger(\mathbf{k}, \omega) + \sigma_x \Im_T^\dagger(\mathbf{k}, \omega)$, the charge carrier self-energies $\Sigma_1^{(\text{h})}(\mathbf{k}, \omega) = \Sigma_{1L}^{(\text{h})}(\mathbf{k}, \omega) + \sigma_x \Sigma_{1T}^{(\text{h})}(\mathbf{k}, \omega)$ and $\Sigma_2^{(\text{h})}(\mathbf{k}, \omega) = \Sigma_{2L}^{(\text{h})}(\mathbf{k}, \omega) + \sigma_x \Sigma_{2T}^{(\text{h})}(\mathbf{k}, \omega)$ in the particle-hole and particle-particle channels, respectively, while the mean-field (MF) charge carrier normal Green's function $g^{(0)}(\mathbf{k}, \omega) = g_L^{(0)}(\mathbf{k}, \omega) + \sigma_x g_T^{(0)}(\mathbf{k}, \omega)$, with the corresponding longitudinal and transverse parts have been obtained as^{8,16},

$$g_L^{(0)}(\mathbf{k}, \omega) = \frac{1}{2} \sum_{\alpha=1,2} \frac{1}{\omega - \xi_{\alpha\mathbf{k}}}, \quad (6)$$

$$g_T^{(0)}(\mathbf{k}, \omega) = \frac{1}{2} \sum_{\alpha=1,2} (-1)^{\alpha+1} \frac{1}{\omega - \xi_{\alpha\mathbf{k}}}, \quad (7)$$

respectively, where $\alpha = 1, 2$, the MF charge carrier spectrum $\xi_{\alpha\mathbf{k}} = Zt\chi_1\gamma_{\mathbf{k}} - Zt'\chi_2\gamma'_{\mathbf{k}} - \mu + (-1)^{\alpha+1}\chi_\perp t_\perp(\mathbf{k})$, the spin correlation function $\chi_2 = \langle S_{ia}^+ S_{i+\hat{\tau}a}^- \rangle$, $\gamma_{\mathbf{k}} =$

$(1/Z) \sum_{\hat{\eta}} e^{i\mathbf{k} \cdot \hat{\eta}}$, $\gamma'_{\mathbf{k}} = (1/Z) \sum_{\hat{\tau}} e^{i\mathbf{k} \cdot \hat{\tau}}$, and Z is the number of the nearest neighbor or next nearest neighbor sites. However, in the bilayer coupling case, the more appropriate classification is in terms of the normal and anomalous Green's functions within the basis of the bonding and antibonding components, i.e., the full charge carrier normal and anomalous Green's functions can be rewritten in the bonding-antibonding representation as,

$$g_{\nu}(\mathbf{k}, \omega) = g_L(\mathbf{k}, \omega) + (-1)^{\nu+1} g_T(\mathbf{k}, \omega), \quad (8a)$$

$$\mathfrak{G}_{\nu}^{\dagger}(\mathbf{k}, \omega) = \mathfrak{G}_L^{\dagger}(\mathbf{k}, \omega) + (-1)^{\nu+1} \mathfrak{G}_T^{\dagger}(\mathbf{k}, \omega), \quad (8b)$$

respectively, where $\nu = 1, 2$, with $\nu = 1$ ($\nu = 2$) represents the corresponding bonding (antibonding) component, then the bonding and antibonding components of the self-energies $\Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega)$ and $\Sigma_{2\nu}^{(h)}(\mathbf{k}, \omega)$ can be obtained from the spin bubble as^{8,16},

$$\begin{aligned} \Sigma_{1\nu}^{(h)}(\mathbf{k}, i\omega_n) &= \Sigma_{1L}^{(h)}(\mathbf{k}, i\omega_n) + (-1)^{\nu+1} \Sigma_{1T}^{(h)}(\mathbf{k}, i\omega_n) \\ &= \frac{1}{8N^2} \sum_{\mathbf{p}\mathbf{q}} \sum_{\nu_1\nu_2\nu_3} \Lambda_{\mathbf{p}+\mathbf{q}+\mathbf{k}}^{\nu\nu_1\nu_2\nu_3} \frac{1}{\beta} \sum_{ip_m} \\ &\quad \times g_{\nu_1}(\mathbf{p} + \mathbf{k}, ip_m + i\omega_n) \Pi_{\nu_2\nu_3}(\mathbf{p}, \mathbf{q}, ip_m), \end{aligned} \quad (9a)$$

$$\begin{aligned} \Sigma_{2\nu}^{(h)}(\mathbf{k}, i\omega_n) &= \Sigma_{2L}^{(h)}(\mathbf{k}, i\omega_n) + (-1)^{\nu+1} \Sigma_{2T}^{(h)}(\mathbf{k}, i\omega_n) \\ &= \frac{1}{8N^2} \sum_{\mathbf{p}\mathbf{q}} \sum_{\nu_1\nu_2\nu_3} \Lambda_{\mathbf{p}+\mathbf{q}+\mathbf{k}}^{\nu\nu_1\nu_2\nu_3} \frac{1}{\beta} \sum_{ip_m} \\ &\quad \times \mathfrak{G}_{\nu_1}^{\dagger}(-\mathbf{p} - \mathbf{k}, -ip_m - i\omega_n) \Pi_{\nu_2\nu_3}(\mathbf{p}, \mathbf{q}, ip_m), \end{aligned} \quad (9b)$$

respectively, with $\Lambda_{\mathbf{k}}^{\nu\nu_1\nu_2\nu_3} = [1 + (-1)^{\nu+\nu_1+\nu_2+\nu_3}] \times [Z(t\gamma_{\mathbf{k}} - t'\gamma'_{\mathbf{k}}) + (-1)^{\nu+\nu_3} t_{\perp}(\mathbf{k})]^2$, and the spin bubble,

$$\begin{aligned} \Pi_{\nu_2\nu_3}(\mathbf{p}, \mathbf{q}, ip_m) &= \frac{1}{\beta} \sum_{iq_m} D_{\nu_2}^{(0)}(\mathbf{q}, iq_m) \\ &\quad \times D_{\nu_3}^{(0)}(\mathbf{q} + \mathbf{p}, iq_m + ip_m), \end{aligned} \quad (10)$$

where the MF spin Green's functions $D_{\nu}^{(0)}(\mathbf{p}, ip_m) = B_{\nu\mathbf{p}}/[(ip_m)^2 - \omega_{\nu\mathbf{p}}^2]$, with the MF spin excitation spectrum $\omega_{\nu\mathbf{p}}$ and function $B_{\nu\mathbf{p}}$ have been given in Ref.^{8,16}.

As in the single layer case¹¹, the pairing force and charge carrier pair gap are incorporated into the self-energy $\Sigma_{2\nu}^{(h)}(\mathbf{k}, \omega)$, then it is called as the effective charge carrier pair gap $\bar{\Delta}_h^{(\nu)}(\mathbf{k}, \omega) = \Sigma_{2\nu}^{(h)}(\mathbf{k}, \omega)$. On the other hand, the self-energy $\Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega)$ renormalizes the MF charge carrier spectrum^{8,16}. Moreover, $\Sigma_{2\nu}^{(h)}(\mathbf{k}, \omega)$ is an even function of ω , while $\Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega)$ is not. For a convenience, $\Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega)$ can be broken up into its symmetric and antisymmetric parts as, $\Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega) = \Sigma_{1\nu e}^{(h)}(\mathbf{k}, \omega) + \omega \Sigma_{1\nu o}^{(h)}(\mathbf{k}, \omega)$, then both $\Sigma_{1\nu e}^{(h)}(\mathbf{k}, \omega)$ and $\Sigma_{1\nu o}^{(h)}(\mathbf{k}, \omega)$ are an even function of ω . As in the conventional superconductors¹⁷, the retarded function $\text{Re}\Sigma_{1\nu e}^{(h)}(\mathbf{k}, \omega)$ may be a constant, independent of (\mathbf{k}, ω) . It just renormalizes the chemical potential, and therefore can be neglected. Now we define the charge carrier coherent weight as $Z_{\text{hF}}^{(\nu)-1}(\mathbf{k}, \omega) = 1 - \text{Re}\Sigma_{1\nu o}^{(h)}(\mathbf{k}, \omega)$, and then in the static limit approximation, i.e., $Z_{\text{hF}}^{(\nu)-1} = 1 - \text{Re}\Sigma_{1\nu o}^{(h)}(\mathbf{k}, \omega = 0) |_{\mathbf{k}=[\pi,0]}$, and $\bar{\Delta}_h^{(\nu)}(\mathbf{k}) = \Sigma_{2\nu}^{(h)}(\mathbf{k}, \omega = 0) = \Sigma_{2L}^{(h)}(\mathbf{k}, \omega = 0) + (-1)^{\nu+1} \Sigma_{2T}^{(h)}(\mathbf{k}, \omega = 0) = \bar{\Delta}_{\text{hL}}(\mathbf{k}) +$

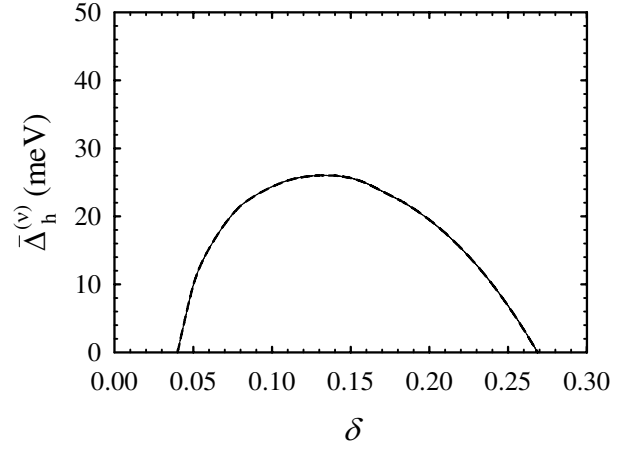


FIG. 1: The bonding (solid line) and antibonding (dashed line) components of the effective charge carrier pair gap parameter as a function of doping for temperature $T = 0.002J$ with parameters $t/J = 2.5$, $t'/t = 0.3$, $t_{\perp}/t = 0.35$ and $J = 110$ meV.

$(-1)^{\nu+1} \bar{\Delta}_{\text{hT}}(\mathbf{k})$, with $\bar{\Delta}_{\text{hL}}(\mathbf{k}) = \bar{\Delta}_{\text{hL}} \gamma_{\mathbf{k}}^{(d)}$, $\bar{\Delta}_{\text{hT}}(\mathbf{k}) = \bar{\Delta}_{\text{hT}}$, and $\gamma_{\mathbf{k}}^{(d)} = (\cos k_x - \cos k_y)/2$, we^{8,16} can obtain the full charge carrier normal and anomalous Green's functions of the bilayer cuprate superconductors. In this case, with the help of these full charge carrier normal and anomalous Green's functions, the self-energy $\Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega)$ and effective charge carrier pair gap $\bar{\Delta}_h^{(\nu)}(\mathbf{k})$ in Eq. (9) can be evaluated explicitly as,

$$\begin{aligned} \Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega) &= \frac{1}{N^2} \sum_{\mathbf{p}\mathbf{q}} \sum_{\nu_1\nu_2\nu_3} \sum_{\sigma_1\sigma_2\sigma_3} \Lambda_{\mathbf{p}+\mathbf{q}+\mathbf{k}}^{\nu\nu_1\nu_2\nu_3} \frac{B_{\nu_2\mathbf{q}} B_{\nu_3\mathbf{q}+\mathbf{p}}}{64\omega_{\nu_2\mathbf{q}}^{(\sigma_2)} \omega_{\nu_3\mathbf{q}+\mathbf{p}}^{(\sigma_3)}} \\ &\quad \times \frac{A_{\sigma_1}^{(\nu_1)}(\mathbf{p} + \mathbf{k}) F_{\sigma_1\sigma_2\sigma_3}^{\nu_1\nu_2\nu_3}(\mathbf{p}, \mathbf{q}, \mathbf{k})}{\omega - E_{\text{h}\nu_1\mathbf{p}+\mathbf{k}}^{(\sigma_1)} - \omega_{\nu_2\mathbf{q}}^{(\sigma_2)} + \omega_{\nu_3\mathbf{q}+\mathbf{p}}^{(\sigma_3)}}, \end{aligned} \quad (11a)$$

$$\begin{aligned} \bar{\Delta}_h^{(\nu)}(\mathbf{k}) &= \frac{1}{N^2} \sum_{\mathbf{p}\mathbf{q}} \sum_{\nu_1\nu_2\nu_3} \sum_{\sigma_1\sigma_2\sigma_3} \Lambda_{\mathbf{p}+\mathbf{q}+\mathbf{k}}^{\nu\nu_1\nu_2\nu_3} \frac{B_{\nu_2\mathbf{q}} B_{\nu_3\mathbf{q}+\mathbf{p}}}{64\omega_{\nu_2\mathbf{q}}^{(\sigma_2)} \omega_{\nu_3\mathbf{q}+\mathbf{p}}^{(\sigma_3)}} \\ &\quad \times \frac{\bar{\Delta}_{\text{hZ}}^{(\nu_1)}(\mathbf{p} + \mathbf{k})}{E_{\text{h}\nu_1\mathbf{p}+\mathbf{k}}^{(\sigma_1)}} \frac{F_{\sigma_1\sigma_2\sigma_3}^{\nu_1\nu_2\nu_3}(\mathbf{p}, \mathbf{q}, \mathbf{k})}{E_{\text{h}\nu_1\mathbf{p}+\mathbf{k}}^{(\sigma_1)} + \omega_{\nu_2\mathbf{q}}^{(\sigma_2)} - \omega_{\nu_3\mathbf{q}+\mathbf{p}}^{(\sigma_3)}}, \end{aligned} \quad (11b)$$

where $\sigma_1, \sigma_2, \sigma_3 = 1, 2$, $F_{\sigma_1\sigma_2\sigma_3}^{\nu_1\nu_2\nu_3}(\mathbf{p}, \mathbf{q}, \mathbf{k}) = Z_{\text{hF}}^{(\nu_1)} \{ n_{\text{F}}(E_{\text{h}\nu_1\mathbf{p}+\mathbf{k}}^{(\sigma_1)}) [n_{\text{B}}(\omega_{\nu_2\mathbf{q}}^{(\sigma_2)}) - n_{\text{B}}(\omega_{\nu_3\mathbf{q}+\mathbf{p}}^{(\sigma_3)})] + n_{\text{B}}(\omega_{\nu_3\mathbf{q}+\mathbf{p}}^{(\sigma_3)}) [1 + n_{\text{B}}(\omega_{\nu_2\mathbf{q}}^{(\sigma_2)})] \}$, $A_{\sigma_1}^{(\nu_1)}(\mathbf{k}) = 1 + \bar{\xi}_{\nu_1\mathbf{k}}/E_{\text{h}\nu_1\mathbf{k}}^{(\sigma_1)}$, with $\omega_{\nu\mathbf{p}}^{(1)} = \omega_{\nu\mathbf{p}}$, $\omega_{\nu\mathbf{p}}^{(2)} = -\omega_{\nu\mathbf{p}}$, $E_{\text{h}\nu\mathbf{k}}^{(1)} = E_{\text{h}\nu\mathbf{k}}$, $E_{\text{h}\nu\mathbf{k}}^{(2)} = -E_{\text{h}\nu\mathbf{k}}$, the renormalized charge carrier excitation spectrum $\bar{\xi}_{\nu\mathbf{k}} = Z_{\text{hF}}^{(\nu)} \xi_{\nu\mathbf{k}}$, the renormalized charge carrier pair gap $\bar{\Delta}_h^{(\nu)}(\mathbf{k}) = Z_{\text{hF}}^{(\nu)} \bar{\Delta}_h^{(\nu)}(\mathbf{k})$, the charge carrier quasiparticle spectrum $E_{\text{h}\nu\mathbf{k}} = \sqrt{\bar{\xi}_{\nu\mathbf{k}}^2 + |\bar{\Delta}_{\text{hZ}}^{(\nu)}(\mathbf{k})|^2}$, and $n_{\text{B}}(\omega)$ and $n_{\text{F}}(E)$ are the boson and fermion distribution functions, respectively. In particular, the equations $Z_{\text{hF}}^{(\nu)-1} = 1 - \text{Re}\Sigma_{1\nu o}^{(h)}(\mathbf{k}, \omega = 0) |_{\mathbf{k}=[\pi,0]}$ and $\bar{\Delta}_h^{(\nu)}(\mathbf{k}) = \Sigma_{2\nu}^{(h)}(\mathbf{k}, \omega = 0)$ have been solved self-consistently in combination with other equations^{8,16}, then all order parameters and chemical potential have been obtained by the self-consistent calculation. In Fig. 1, we plot the self-consistently calculated result⁸ of

the effective charge carrier pair gap parameter for the bonding $\bar{\Delta}_h^{(1)}$ (solid line) and antibonding $\bar{\Delta}_h^{(2)}$ (dashed line) components versus the doping concentration in temperature $T = 0.002J$ with parameters $t/J = 2.5$, $t'/t = 0.3$, $t_\perp/t = 0.35$, and $J = 110$ meV. It is shown clearly that the maximal $\bar{\Delta}_h^{(1)}$ and $\bar{\Delta}_h^{(2)}$ occur around the optimal doping, and then decrease in both the underdoped and the overdoped regimes. Moreover, both the bonding and antibonding components of the effective charge carrier pair gap parameter have the same magnitude in a given doping concentration⁸, which implies the transverse part of the charge carrier pair gap $\bar{\Delta}_{hT} \approx 0$, and then $\bar{\Delta}_h^{(1)} \approx \bar{\Delta}_h^{(2)} \approx \bar{\Delta}_{hL}$. This result shows that although there is a single electron *interlayer* coherent hopping (2) in the bilayer cuprate superconductors, the coupling strength for the *interlayer* pairs vanishes, which reflects that within the framework of the kinetic energy driven SC mechanism, the weak charge carrier interaction directly from the *interlayer* coherent hopping (2) in the kinetic energy by exchanging spin excitations does not provide any contribution to the charge carrier pair gap in the particle-particle channel, and then the transverse part of the charge carrier pair gap $\bar{\Delta}_{hT} \approx 0$. This is different from the strong charge carrier interaction directly from the *intralayer* hopping in the kinetic energy by exchanging spin excitations, which induces superconductivity in the particle-particle channel¹¹, and then the charge carrier pair gap is dominated by the corresponding longitudinal part, i.e., $\bar{\Delta}_h^{(1)} \approx \bar{\Delta}_h^{(2)} \approx \bar{\Delta}_{hL}$. This result is also consistent with the experimental results of the bilayer cuprate

superconductor $\text{Bi(Pb)}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ ⁷, where the SC gap separately for the bonding and antibonding components has been measured, and it is found that both the antibonding and bonding components are identical within the experimental uncertainties.

Now we discuss the interplay between the SC-gap and normal-state pseudogap in the bilayer cuprate superconductors. As in the single layer case¹², the self-energy $\Sigma_1^{(h)}(\mathbf{k}, \omega)$ in Eq. (11a) in the particle-hole channel can also be rewritten approximately as,

$$\Sigma_1^{(h)}(\mathbf{k}, \omega) \approx \frac{[2\bar{\Delta}_{pg}(\mathbf{k})]^2}{\omega + M_{\mathbf{k}}}, \quad (12)$$

where $M_{\mathbf{k}} = M_{L\mathbf{k}} + \sigma_x M_{T\mathbf{k}}$ is the energy spectrum of $\Sigma_1^{(h)}(\mathbf{k}, \omega)$. As in the case of the effective charge carrier pair gap, the interaction force and normal-state pseudogap have been incorporated into $\bar{\Delta}_{pg}(\mathbf{k}) = \bar{\Delta}_{pgL}(\mathbf{k}) + \sigma_x \bar{\Delta}_{pgT}(\mathbf{k})$, and therefore it is called as the effective normal-state pseudogap. In the bonding-antibonding representation, the self-energy in Eq. (12) can be expressed as,

$$\Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega) \approx \frac{[2\bar{\Delta}_{pg}^{(\nu)}(\mathbf{k})]^2}{\omega + M_{\nu\mathbf{k}}}, \quad (13)$$

with $M_{\nu\mathbf{k}} = M_{L\mathbf{k}} + (-1)^{\nu+1} M_{T\mathbf{k}}$, and $\bar{\Delta}_{pg}^{(\nu)}(\mathbf{k}) = \bar{\Delta}_{pgL}(\mathbf{k}) + (-1)^{\nu+1} \bar{\Delta}_{pgT}(\mathbf{k})$. Substituting $\Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega)$ in Eq. (13) into Eq. (5), the full charge carrier normal and anomalous Green's functions can be obtained straightforwardly as,

$$\begin{aligned} g_\nu(\mathbf{k}, \omega) &= \frac{1}{\omega - \xi_{\nu\mathbf{k}} - \Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega) - [\bar{\Delta}_h^{(\nu)}(\mathbf{k})]^2 / [\omega + \xi_{\nu\mathbf{k}} + \Sigma_{1\nu}^{(h)}(-\mathbf{k}, -\omega)]} \\ &= \frac{[U_{1hk}^{(\nu)}]^2}{\omega - E_{1hk}^{(\nu)}} + \frac{[V_{1hk}^{(\nu)}]^2}{\omega + E_{1hk}^{(\nu)}} + \frac{[U_{2hk}^{(\nu)}]^2}{\omega - E_{2hk}^{(\nu)}} + \frac{[V_{2hk}^{(\nu)}]^2}{\omega + E_{2hk}^{(\nu)}}, \end{aligned} \quad (14a)$$

$$\begin{aligned} \mathfrak{G}_\nu^\dagger(\mathbf{k}, \omega) &= -\frac{\bar{\Delta}_h^{(\nu)}(\mathbf{k})}{[\omega - \xi_{\nu\mathbf{k}} - \Sigma_{1\nu}^{(h)}(\mathbf{k}, \omega)][\omega + \xi_{\nu\mathbf{k}} + \Sigma_{1\nu}^{(h)}(-\mathbf{k}, -\omega)] - [\bar{\Delta}_h^{(\nu)}(\mathbf{k})]^2} \\ &= -\frac{\alpha_{1\mathbf{k}}^{(\nu)} \bar{\Delta}_h^{(\nu)}(\mathbf{k})}{2E_{1hk}^{(\nu)}} \left(\frac{1}{\omega - E_{1hk}^{(\nu)}} - \frac{1}{\omega + E_{1hk}^{(\nu)}} \right) + \frac{\alpha_{2\mathbf{k}}^{(\nu)} \bar{\Delta}_h^{(\nu)}(\mathbf{k})}{2E_{2hk}^{(\nu)}} \left(\frac{1}{\omega - E_{2hk}^{(\nu)}} - \frac{1}{\omega + E_{2hk}^{(\nu)}} \right), \end{aligned} \quad (14b)$$

respectively, where $\alpha_{1\mathbf{k}}^{(\nu)} = \{[E_{1hk}^{(\nu)}]^2 - M_{\nu\mathbf{k}}^2\} / \{[E_{1hk}^{(\nu)}]^2 - [E_{2hk}^{(\nu)}]^2\}$, $\alpha_{2\mathbf{k}}^{(\nu)} = \{[E_{2hk}^{(\nu)}]^2 - M_{\nu\mathbf{k}}^2\} / \{[E_{1hk}^{(\nu)}]^2 - [E_{2hk}^{(\nu)}]^2\}$, $E_{1hk}^{(\nu)} = \sqrt{(\Omega_{\nu\mathbf{k}} + \Theta_{\nu\mathbf{k}})/2}$, and $E_{2hk}^{(\nu)} = \sqrt{(\Omega_{\nu\mathbf{k}} - \Theta_{\nu\mathbf{k}})/2}$, with the kernel functions,

$$\Omega_{\nu\mathbf{k}} = \xi_{\nu\mathbf{k}}^2 + M_{\nu\mathbf{k}}^2 + 8[\bar{\Delta}_{pg}^{(\nu)}(\mathbf{k})]^2 + [\bar{\Delta}_h^{(\nu)}(\mathbf{k})]^2, \quad (15a)$$

$$\Theta_{\nu\mathbf{k}} = \sqrt{(\xi_{\nu\mathbf{k}}^2 - M_{\nu\mathbf{k}}^2)\beta_{1\mathbf{k}}^{(\nu)} + 16[\bar{\Delta}_{pg}^{(\nu)}(\mathbf{k})]^2\beta_{2\mathbf{k}}^{(\nu)} + [\bar{\Delta}_h^{(\nu)}(\mathbf{k})]^4}, \quad (15b)$$

where $\beta_{1\mathbf{k}}^{(\nu)} = \xi_{\nu\mathbf{k}}^2 - M_{\nu\mathbf{k}}^2 + 2[\bar{\Delta}_h^{(\nu)}(\mathbf{k})]^2$, $\beta_{2\mathbf{k}}^{(\nu)} = (\xi_{\nu\mathbf{k}} -$

$M_{\nu\mathbf{k}})^2 + [\bar{\Delta}_h^{(\nu)}(\mathbf{k})]^2$, while the coherence factors,

$$(U_{1hk}^{(\nu)})^2 = \frac{1}{2} \{ \alpha_{1\mathbf{k}}^{(\nu)} [1 + \frac{\xi_{\nu\mathbf{k}}}{E_{1hk}^{(\nu)}}] - \alpha_{3\mathbf{k}}^{(\nu)} [1 + \frac{M_{\nu\mathbf{k}}}{E_{1hk}^{(\nu)}}] \}, \quad (16a)$$

$$(V_{1hk}^{(\nu)})^2 = \frac{1}{2} \{ \alpha_{1\mathbf{k}}^{(\nu)} [1 - \frac{\xi_{\nu\mathbf{k}}}{E_{1hk}^{(\nu)}}] - \alpha_{3\mathbf{k}}^{(\nu)} [1 - \frac{M_{\nu\mathbf{k}}}{E_{1hk}^{(\nu)}}] \}, \quad (16b)$$

$$(U_{2hk}^{(\nu)})^2 = -\frac{1}{2} \{ \alpha_{2\mathbf{k}}^{(\nu)} [1 + \frac{\xi_{\nu\mathbf{k}}}{E_{2hk}^{(\nu)}}] - \alpha_{3\mathbf{k}}^{(\nu)} [1 + \frac{M_{\nu\mathbf{k}}}{E_{2hk}^{(\nu)}}] \}, \quad (16c)$$

$$(V_{2hk}^{(\nu)})^2 = -\frac{1}{2} \{ \alpha_{2\mathbf{k}}^{(\nu)} [1 - \frac{\xi_{\nu\mathbf{k}}}{E_{2hk}^{(\nu)}}] - \alpha_{3\mathbf{k}}^{(\nu)} [1 - \frac{M_{\nu\mathbf{k}}}{E_{2hk}^{(\nu)}}] \}, \quad (16d)$$

satisfy the sum rule: $[U_{1\mathbf{hk}}^{(\nu)}]^2 + [V_{1\mathbf{hk}}^{(\nu)}]^2 + [U_{2\mathbf{hk}}^{(\nu)}]^2 + [V_{2\mathbf{hk}}^{(\nu)}]^2 = 1$, with $\alpha_{3\mathbf{k}}^{(\nu)} = [2\bar{\Delta}_{\text{pg}}^{(\nu)}(\mathbf{k})]^2 / \{[E_{1\mathbf{hk}}^{(\nu)}]^2 - [E_{2\mathbf{hk}}^{(\nu)}]^2\}$, and then the corresponding effective normal-state pseudogap $\bar{\Delta}_{\text{pg}}^{(\nu)}(\mathbf{k})$ and energy spectra $M_{\nu\mathbf{k}}$ can be obtained explicitly in terms of the self-energies $\Sigma_{1\nu}^{(\text{h})}(\mathbf{k}, \omega)$ in Eq. (11a) as,

$$\bar{\Delta}_{\text{pg}}^{(\nu)}(\mathbf{k}) = \frac{L_2^{(\nu)}(\mathbf{k})}{2\sqrt{L_1^{(\nu)}(\mathbf{k})}}, \quad (17a)$$

$$M_{\nu\mathbf{k}} = \frac{L_2^{(\nu)}(\mathbf{k})}{L_1^{(\nu)}(\mathbf{k})}, \quad (17b)$$

where $L_1^{(\nu)}(\mathbf{k})$ and $L_2^{(\nu)}(\mathbf{k})$ are obtained from Eq. (11a) as,

$$L_1^{(\nu)}(\mathbf{k}) = \frac{1}{N^2} \sum_{\mathbf{pq}} \sum_{\nu_1\nu_2\nu_3} \sum_{\sigma_1\sigma_2\sigma_3} \Lambda_{\mathbf{p}+\mathbf{q}+\mathbf{k}}^{\nu\nu_1\nu_2\nu_3} \frac{B_{\nu_2\mathbf{q}}B_{\nu_3\mathbf{q}+\mathbf{p}}}{64\omega_{\nu_2\mathbf{q}}^{(\sigma_2)}\omega_{\nu_3\mathbf{q}+\mathbf{p}}^{(\sigma_3)}} \times \frac{A_{\sigma_1}^{(\nu_1)}(\mathbf{p}+\mathbf{k})F_{\sigma_1\sigma_2\sigma_3}^{\nu\nu_1\nu_2\nu_3}(\mathbf{p}, \mathbf{q}, \mathbf{k})}{(\omega_{\nu_3\mathbf{q}+\mathbf{p}}^{(\sigma_3)} - E_{\text{h}\nu_1\mathbf{p}+\mathbf{k}}^{(\sigma_1)} - \omega_{\nu_2\mathbf{q}}^{(\sigma_2)})^2}, \quad (18a)$$

$$L_2^{(\nu)}(\mathbf{k}) = \frac{1}{N^2} \sum_{\mathbf{pq}} \sum_{\nu_1\nu_2\nu_3} \sum_{\sigma_1\sigma_2\sigma_3} \Lambda_{\mathbf{p}+\mathbf{q}+\mathbf{k}}^{\nu\nu_1\nu_2\nu_3} \frac{B_{\nu_2\mathbf{q}}B_{\nu_3\mathbf{q}+\mathbf{p}}}{64\omega_{\nu_2\mathbf{q}}^{(\sigma_2)}\omega_{\nu_3\mathbf{q}+\mathbf{p}}^{(\sigma_3)}} \times \frac{A_{\sigma_1}^{(\nu_1)}(\mathbf{p}+\mathbf{k})F_{\sigma_1\sigma_2\sigma_3}^{\nu\nu_1\nu_2\nu_3}(\mathbf{p}, \mathbf{q}, \mathbf{k})}{\omega_{\nu_3\mathbf{q}+\mathbf{p}}^{(\sigma_3)} - E_{\text{h}\nu_1\mathbf{p}+\mathbf{k}}^{(\sigma_1)} - \omega_{\nu_2\mathbf{q}}^{(\sigma_2)}}. \quad (18b)$$

Now we obtain the effective normal-state pseudogap parameter from Eq. (17a) as,

$$\bar{\Delta}_{\text{pg}}^{(\nu)} = \frac{1}{N} \sum_{\mathbf{k}} \bar{\Delta}_{\text{pg}}^{(\nu)}(\mathbf{k}), \quad (19)$$

where $\nu = 1, 2$, with $\bar{\Delta}_{\text{pg}}^{(1)}$ and $\bar{\Delta}_{\text{pg}}^{(2)}$ are the corresponding bonding and antibonding components of the effective normal-state pseudogap parameter, respectively.

In Fig. 2, we plot the bonding ($2\bar{\Delta}_{\text{pg}}^{(1)}$) (dotted line) and antibonding ($2\bar{\Delta}_{\text{pg}}^{(2)}$) (solid line) components of the effective normal-state pseudogap parameter, and the effective charge carrier pair gap parameter ($2\bar{\Delta}_{\text{hL}}$) (dashed line) as a function of doping for $T = 0.002J$ with $t/J = 2.5$, $t'/t = 0.3$, $t_{\perp}/t = 0.35$ and $J = 110$ meV in comparison with the corresponding experimental data² observed on different families of the cuprate superconductors (inset). Obviously, both $\bar{\Delta}_{\text{pg}}^{(1)}$ and $\bar{\Delta}_{\text{pg}}^{(2)}$ have almost the same magnitude in a given doping concentration, which implies the transverse part of the effective normal-state pseudogap parameter $\bar{\Delta}_{\text{pgT}} \approx 0$ and $\bar{\Delta}_{\text{pg}}^{(1)} \approx \bar{\Delta}_{\text{pg}}^{(2)} \approx \bar{\Delta}_{\text{pgL}}$, then in analogy to the single layer case¹², the two-gap feature observed on the bilayer cuprate superconductors² is qualitatively reproduced. Moreover, the effective normal-state pseudogap parameter $\bar{\Delta}_{\text{pgL}}$ is much larger than the effective charge carrier pair gap parameter $\bar{\Delta}_{\text{hL}}$ in the underdoped regime, then it smoothly decreases with increasing the doping concentration. In particular, both $\bar{\Delta}_{\text{pgL}}$ and $\bar{\Delta}_{\text{hL}}$ converge to the end of the SC dome. The present results also show that the weak charge carrier interaction directly from the *interlayer* coherent hopping (2) in the kinetic energy by exchanging spin excitations

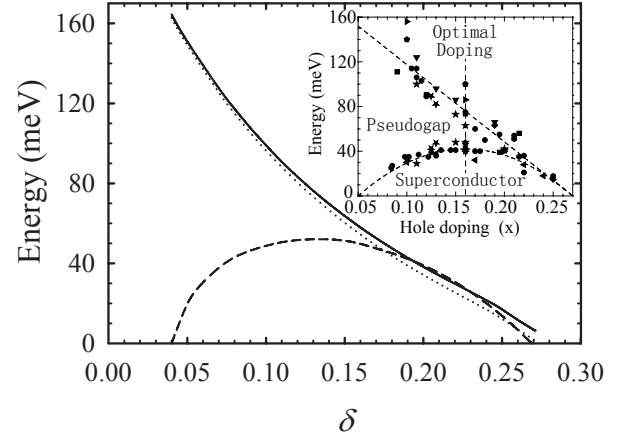


FIG. 2: The bonding ($2\bar{\Delta}_{\text{pg}}^{(1)}$) (dotted line) and antibonding ($2\bar{\Delta}_{\text{pg}}^{(2)}$) (solid line) components of the effective normal-state pseudogap parameter, and the effective charge carrier pair gap parameter ($2\bar{\Delta}_{\text{hL}}$) (dashed line) as a function of doping for temperature $T = 0.002J$ with parameters $t/J = 2.5$, $t'/t = 0.3$, $t_{\perp}/t = 0.35$ and $J = 110$ meV. Inset: the experimental data observed on different families of the cuprate superconductors taken from Ref. 2.

does not provide the contribution to the effective normal-state pseudogap in the particle-hole channel, and then the transverse part of the effective normal-state pseudogap parameter $\bar{\Delta}_{\text{pgT}} \approx 0$. However, in contrast to this case, the strong charge carrier interaction directly from the *intralayer* hopping in the kinetic energy by exchanging spin excitations therefore can induce the normal-state pseudogap in the particle-hole channel¹², and then the normal-state pseudogap is dominated by the corresponding longitudinal part, i.e., $\bar{\Delta}_{\text{pg}}^{(1)} \approx \bar{\Delta}_{\text{pg}}^{(2)} \approx \bar{\Delta}_{\text{pgL}}$. This result is also consistent with the experimental results of the bilayer cuprate superconductors², since only one normal-state pseudogap is observed in the bilayer cuprate superconductors by using different measurement techniques².

The essential physics of the two-gap feature in the bilayer cuprate superconductors is the same as in the single layer case¹², and can be attributed to the doping and temperature dependence of the charge carrier interactions in the particle-hole and particle-particle channels directly from the kinetic energy by exchanging spin excitations. Our present results also indicate that although BS due to the presence of the *interlayer* coherent hopping (2) causes a peak-dip-hump structure in the low-energy excitation spectrum of the bilayer cuprate superconductors^{8,16}, it may have not an impact on the overall global features for the SC gap and normal-state pseudogap. Furthermore, in the present bilayer case, we have also calculated the doping dependence of the coupling strength V_{eff} , and the result shows that as in the single layer case¹², the coupling strength V_{eff} smoothly decreases upon increasing the doping concentration from a strong-coupling case in the underdoped regime to a weak-coupling side in the overdoped regime. Since the charge carrier interactions in both the particle-hole and particle-particle channels are mediated by the same spin excitations as shown in Eq. (9), therefore all these charge

carrier interactions are controlled by the same magnetic interaction J . In this sense, both the normal-state pseudogap and SC gap in the phase diagram of the bilayer cuprate superconductors are dominated by one energy scale. This is why both $\bar{\Delta}_{\text{pgT}} \approx 0$ (then $\bar{\Delta}_{\text{pg}}^{(1)} \approx \bar{\Delta}_{\text{pg}}^{(2)} \approx \bar{\Delta}_{\text{pgL}}$) and $\bar{\Delta}_{\text{hT}} \approx 0$ (then $\bar{\Delta}_{\text{h}}^{(1)} \approx \bar{\Delta}_{\text{h}}^{(2)} \approx \bar{\Delta}_{\text{hL}}$) simultaneously in the bilayer cuprate superconductors, and then the two-gap behavior is a universal feature for the single layer and bilayer cuprate superconductors.

In conclusion, we have discussed the interplay between the SC gap and normal-state pseudogap in the bilayer cuprate superconductors based on the framework of the kinetic energy driven SC mechanism. Our results show that the two-gap behavior is a universal feature for the single layer and bilayer cuprate superconductors. The weak charge carrier interaction directly from the *inter-layer* coherent hopping (2) in the kinetic energy by exchanging spin excitations does not provide the contribution to the normal-state pseudogap in the particle-hole channel and charge carrier pair gap in the particle-particle channel, which leads to that the transverse parts of the effective normal-state pseudogap parameter $\bar{\Delta}_{\text{pgT}} \approx 0$ and effective charge carrier pair gap parameter $\bar{\Delta}_{\text{hT}} \approx 0$ simultaneously, while only the strong charge

carrier interaction directly from the *intralayer* hopping in the kinetic energy by exchanging spin excitations therefore induces the normal-state pseudogap in the particle-hole channel and charge carrier pair gap in the particle-particle channel, and then the normal-state pseudogap and charge carrier pair gap are dominated by the corresponding longitudinal parts, i.e., $\bar{\Delta}_{\text{pg}}^{(1)} \approx \bar{\Delta}_{\text{pg}}^{(2)} \approx \bar{\Delta}_{\text{pgL}}$ and $\bar{\Delta}_{\text{h}}^{(1)} \approx \bar{\Delta}_{\text{h}}^{(2)} \approx \bar{\Delta}_{\text{hL}}$.

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